2021 IEEE 17th International Conference on Automation Science and Engineering (CASE)
August 23-27, 2021. Lyon, France

2021 IEEE 17th International Conference on Automation Science and Engineering (CASE) | 978-1-6654-1873-7/21/$31.00 ©2021 IEEE | DOI: 10.1109/CASE49439.2021.9551461

Smooth Static Walking for Quadruped Robots based on the Lemniscate of Gerono

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Abstract—**In the research of quadruped robots, stability is a very important consideration for gait design. When the robots have symmetrical structure, stability can be easily guaranteed. However, when the robots are carrying some additional devices or payloads unevenly, the position of the center of gravity (COG) may deviate from the geometrical center, which makes it a challenging task to guarantee stability. To handle this, it is of great significance to improve the stability margin during gait design. To this end, a smooth static walking gait with the maximum stability margin is developed in this paper. An algorithm of COG trajectory optimization based on the lemniscate of Gerono is proposed. The advantage of this algorithm is that the COG trajectory is smooth and continuous at any order, which avoids abrupt changes in velocity or acceleration of the robot during walking. The two parameters in the lemniscate are the main tuning parameters. According to the size of the robot, the algorithm can automatically calculate the optimal parameters (adjust the shape of the Gerono lemniscate curve) and balance the relationship between the step size and the stability margin during the robot movement. Simulation results demonstrate the effectiveness of the proposed method, and we use a mass block experiment to prove the insensitivity of the gait algorithm to the position of the COG.**

I. Introduction

The research of legged robots has attracted extensive attention in the field of mobile robots in recent years. This may because legged robots are much more adaptable to complex terrains than wheeled and track-based robots. Legged robots mainly include one-legged robots, biped robots, quadruped robots, hexapod robots, and so on. Among them, quadruped robots seem to be more popular than the other legged robots since it makes a good balance between stability and complexity. Among all the research areas of quadruped robot, gait planning is a significant aspect since the robots cannot walk through various terrains or accomplish specific tasks without robust walking gaits.

*This work was supported by National Natural Science Foundation of China (62003188, 61803221, and U1813216), Guangdong Special Branch Plan for Young Talent with Scientific and Technological Innovation (2019TQ05Z111).

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Quadruped gaits can be classified into static gaits and dynamic gaits. No matter what type of gait, maintaining stability of the robot is the key problem for gait planning. In a static gait, the robot has at least three legs in contact with the ground at any moment. It is well known that a walking robot is statically stable if the vertical projection of the COG of the robot is within the support polygon. Meanwhile, the concept of stability margin [1] is an important index to evaluate the performance of a gait algorithm. A detailed mathematical analysis given by McGhee and Frank [1] shows that there exists a unique optimum gait sequence that maximizes the static stability. Besides, the stability margin is mainly influenced by the COG trajectory. Therefore, how to plan the trajectory of the COG becomes a key problem to obtain enough stability margin for the robot.

After McGhee and Frank coming up with the Stability Margin (SM) [1], various stability criteria for the static gaits were proposed，for example, Longitudinal Stability Margin (LSM) [2] given by S.M. Song and K.J. Waldron, Static Stability Margin (SSM) [3] proposed by C. D. Zhang and S. M. Song, Energy Stability Margin (ESM) [4] and the Normalized Energy Margin (NESM) [5] and so on. Among these stability criteria, researchers prefer to use the SM more often because it's more intuitive and computationally easier than the other methods. Therefore, we also use SM to evaluate the stability of the robot and calculate the stability margin in this paper.

The stability margin is determined by the projection location of the quadruped robot's COG in the supporting polygon. To optimize the stability margin, it requires a careful planning for the trajectory of the COG. A great deal of researches had been done in this area. Fan-Tian Cheng et al. [6] might be the first to point out that the robot's body sway motion could greatly increase the stability margin of the robot. They also proposed two sway motion, including Y-Sway and E-Sway, which provides a new idea for the COG trajectory planning. These two sway motions were easy to implement. By using these two sway motions, the stability margin of quadruped robots can be greatly improved. In [7], B. H. Kim et al. identified the centroid of foot polygons formed in every step of a moving quadruped robot, analyzed and put forward a performance index to measure the balance of motion. According to the robot's centroid trajectory and the proposed balance index, a kind of swing motion which is important in quadruped motion is estimated. Dimitris Pongas et al. designed a special COG trajectory algorithm that allows the robot to traverse rough terrains [8]. They added sinusoidal components to the fore-aft and lateral motion of the robot's body. The experimental results were nice. In [9], a novel line-based COG trajectory planner was introduced. The algorithm was simpler than traditional polygon-based methods. Besides, there are also other ways to improve the stability margin. Dong-Oh Kang et al. proposed a new criterion for the stability margin based on ZMP in response to unknown forces [10]. T.T. Lee gives the mathematical expression of the relationship among the stability margin, step size, and duty factor [11].

However, most of the aforementioned work only consider to obtain an acceptable stability margin rather than maximize the stability margin of the robot. Besides, most of the gait algorithms have not been tested on quadruped robots with COG position significantly deviated from its geometric center. The robot can walk normally with a certain stability margin, but if a heavy object is placed on the corner of of the body, the deviated COG position may get out of the stability margin and the robot may tip over. In this case, it is important to design a more robust gait algorithm that can ensure a large enough stability margin which is insensitive to the shifting of the robot's COG.

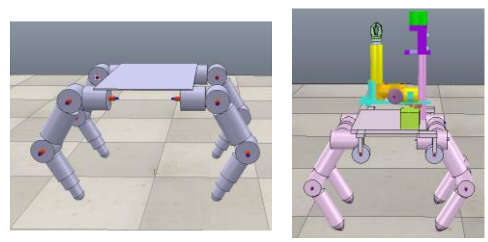
In this paper, a new COG trajectory planning method was proposed for quadruped robots based on the lemniscate of Gerono, which ensures the robot to have a large stability margin at every moment of movement. And the gait algorithm can be easily applied to robots with different sizes by changing the parameters. It is shown that the proposed gait can effectively improve the stability margin of a quadruped robot. Several factors that influence the gait stability margin of the robot during static walking are analyzed, including two parameters in the lemniscate curve and the step length. Optimal values for those parameters are computed. The robustness of the gait is verified through both V-rep simulations and experiments on our lab robot. The main contributions of this paper are as follows: (1) Taking stability as the primary factor, a new gait algorithm of quadruped robot is designed (It includes the design of the COG trajectory based on a special lemniscate, the design of six stages of periodic motion and the periodic coordination planning between the body and the foot). This gait not only enables the quadruped robot with symmetrical structure to have as much stability margin as possible at any moment of movement, but also enables the quadruped robot with the COG deviated from geometric center to have excellent stability (i.e., insensitive to the COG shifting). (2) The proposed method has easy-to-tune parameters, which makes it widely applicable to different quadruped robots. As long as the basic dimensions of the robot are given, the algorithm can be applied to the robot easily. The algorithm has strong robustness and can obviously improve the stability margin. This allows the robot to perform specific tasks (tasks that require high stability) in this gait.

This rest of the paper is arranged according to the following structure. Section II introduces the design of a smooth static gait based on the Gerono lemniscate and addresses how the walking cycle is coordinated with the COG trajectory. In Section III, the relationship of the gait parameters and stability margin is analyzed to find the optimal parameters according to the real robot's size. Section IV gives the simulation results. Conclusions are given in Section V.

II. Smooth Static Gait Design

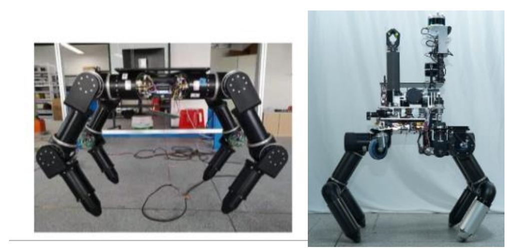
A. Quadruped Robot Models

We built two robot models used in this paper which are shown in Fig. 1. They are simulation models established based on the real robots in our laboratory. The real robots are shown in Fig. 2. The left robot which we call Robot 1 in this paper consists mainly of a body and four identical legs connected to the body. Each leg has three joints, including the hip abduction/adduction (HAA), hip flexion/extension (HFE), and knee flexion/extension (KFE). We do the gait algorithm's basic design and verification mainly on this symmetrical robot. The robot on the right is a simulation model based on the Robot 2 which has attached with many devices and is more complicated. We mainly use it to verify that the gait algorithm can adapt to different quadruped robots and is insensitive to the shift of COG position (Robot 2 is not completely symmetrical).



(a) Robot 1 (b) Robot 2

Fig. 1. The simulation quadruped robots in V-rep



(a) Robot 1 (b) Robot 2

Fig. 2. The prototype quadruped robots in the lab.

To simplify the problem, we make the following assumptions for Robot 1:

(1) Assume that the center of mass lies in the geometric center of the body.

(2) The height of the robot remains unchanged during movement.

B. Smooth COG trajectory

Static gait has six possible leg sequences, among which the sequence of [right hind, right front, left hind, left front] (RH, RF, LH, LF) is proved to have the optimal static stability margin [12]. Therefore, this sequence of legs is adopted in this paper. In general, the gait cycle of a quadruped robot can be divided into eight phases as shown in Fig. 3, four of which are the four-leg standing phases and four are the one-leg swinging phases.

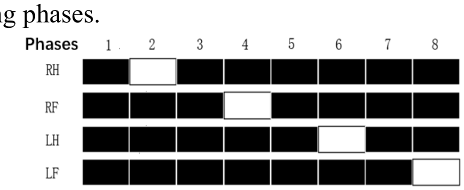


Fig.3. The gait diagram of a quadruped robot. The black box represents the leg in contact with the ground, and the white box represents the swing leg.

According to the stability criterion, in a static gait, the COG should fall in the polygon formed by the supporting feet to remain stable. This condition can be easily satisfied when all the four feet of the robot touch the ground. More considerations are needed during the one-leg swinging phase to make the COG projection of the robot stay in the supporting triangle and have enough stability margin. With the swing leg sequence of 4-2-3-1, the supporting triangles during the one-leg swinging phase are depicted in Fig. 4.

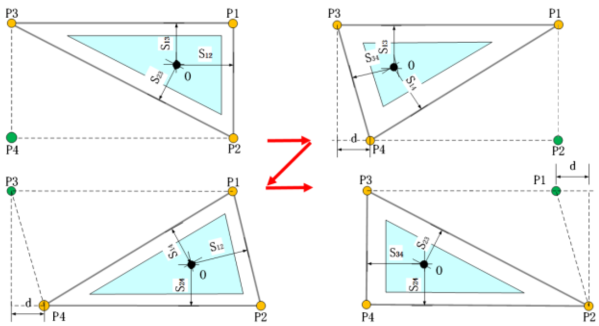


Fig.4. The supporting triangle changes in a gait cycle.

In Fig. 4, the yellow circles represent the feet that touch the ground and the green one represents the swinging foot. The blue areas are contracted triangles within the supporting polygon. The COG falling in the contracted triangle can ensure the stability of the robot even if there are some uncertainties and measurement deviations. d is the step length of the robot. S_{ij} represents the distance from the projected COG to the line connecting the i-th foot and the j-th foot.

To maintain stability, a simple rule can be found, that is, the COG should always be placed in the opposite diagonal position of the swing leg. For example, when the right-hind leg swings, the COG should be projected in left-front, and when the right-front leg swings, the COG should be located in the left-hind. So is the case when the other legs swing.

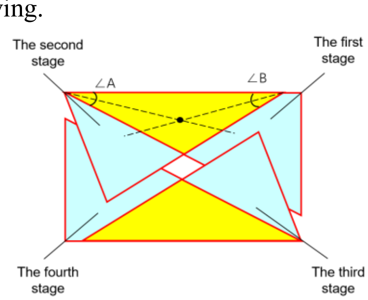


Fig.5. Four stages of COG movement within the supporting triangle.

The movement of the COG within the supporting polygon can be divided into four stages as shown in Fig. 5. It can be seen that the supporting triangles of the first and second stages have an overlapped region, shown as the yellow part, which is usually called double support triangle (DST) [13]. Therefore, we can remove the third phase (four-legged standing phase) in Fig. 3. Similarly, phase 7 in Fig. 3 can also be removed. However, it can be found that there is no overlap between the second and the third stages in Fig. 5. So we have to add a four-leg stance phase between them, which is phase 5 in Fig. 3. For the same reason, add the first phase in Fig. 3. As a result, the new gait sequence for a cycle is shown in Fig. 6.

Fig. 6. The new gait diagram of the quadruped robot.

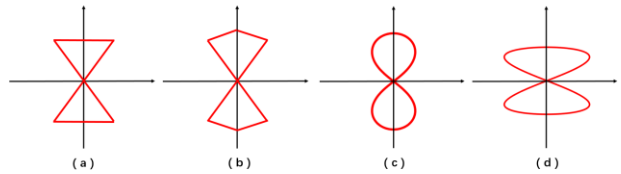


Fig. 7. Several types of figure-eight curves.

Also, it can be observed from Fig. 5 that the trajectory of the COG should follow a figure-eight curve to ensure the maximum stability margin of the quadruped robot in the process of moving. For the figure-eight curve, there are multiple options as shown in Fig. 7. In Fig.7 (a), simple line trajectories are used as COG's trajectory. However, it has discontinuous velocity and acceleration, which inevitably degrades the walking performance of the robot. We can also design the trajectory as the shape shown in Fig. 7 (b). The inclination of the two-line segments on the top of the curve lies in the middle of angles A and B in Fig. 5, that is, the trajectory of the COG passes through DST along the angular bisector of angles A and B, which makes the stability margin better. However, it's velocity is still discontinuous.

A better option is to choose a smooth curve as the COG trajectory. A curve with continuity to the third order can guarantee a continuous velocity and acceleration of motion, which greatly increases the smoothness of the movement and thus improves the walking performance for the robot. Two kinds of smooth curves are investigated here, including the lemniscate of Bernoulli shown in Fig.7 (c) and the lemniscate of Gerono in Fig.7 (d).

The parametric equation for the lemniscate of Bernoulli is as follows [14]:

$$\begin{aligned} x &= \frac{a\sin t}{1+\cos^2 t} \\ y &= \frac{a\sin t\cos t}{1+\cos^2 t} \end{aligned} \tag{1}$$

Here if we replace the coefficient a in the x and y equation with two different coefficients a and b, then we can adjust the amplitude in the x and y directions separately. We call it the extended form for the lemniscate of Bernoulli

$$x = \frac{a\sin t}{1+\cos^2 t}$$
$$y = \frac{b\sin t\cos t}{1+\cos^2 t} \tag{2}$$

Similarly, the parametric equation for the lemniscate of Gerono (also in extended form) is given as follows [15]:

$$x = a\sin t$$
$$y = b\sin 2t \tag{3}$$

Comparing (2) and (3), we finally choose the lemniscate of Gerono as the COG trajectory. There are two reasons. First, the form of the parametric equation is simpler. Second and more importantly, the amplitudes of the lemniscate of Gerono in the x and y directions are a and b, which is convenient for design and control while the lemniscate of Bernoulli is not (The amplitude in the y-direction is not b).

The COG trajectory also needs to take into account the gait cycle period T and the step length d . Assume that the walking direction is along the y-axis, then the trajectory for the COG is designed as follows:

$$x = a\sin(\frac{2\pi}{T}t)$$
$$y = b\sin(\frac{4\pi}{T}t) + d\frac{t}{T} \tag{4}$$

C. Coordination of the COG motion with foot swing

Next, to make the robot have a complete gait, we need to carry out periodic consistent planning for the trajectory of the COG and the trajectory of the foot. Fig. 8 shows the leg swing events during the COG moving.

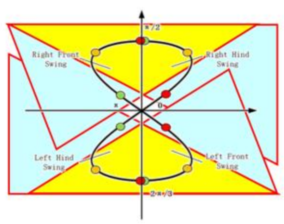


Fig.8. Leg swing events during the COG moving.

Leg swing consists of a lift-off phase (From the red circle to the yellow circle) and a touch-down phase (From the yellow circle to the green circle). Because of DST, the right-front foot leaves the ground immediately when the right-hind foot touches the ground. The left-front foot and left-hind foot are the same. This process is represented in the figure as overlapping green and red circles at $\frac{T}{4}$ and $\frac{3T}{4}$. But in the transition from the right-front foot to the left-hind foot and the left-front foot to the right-back foot, we must add a four-leg stance phase to maintain stability. That is the stage from the green circle to the red circle in the figure (not overlap).

When comparing the swing phase of the leg with the supporting triangle shown in Fig. 4, it can be found that the COG is in the triangle. At the same time, when the right-hind foot transitions to the right-front foot and the left-hind foot transitions to the left-front foot, the COG is in the DST and the robot is stable. Therefore, the whole moving process of the robot is stable and has a large stability margin (to be analyzed in detail in the next section).

D. The trajectories of the four legs

The swing trajectory of each leg is a smooth sinusoidal curve. In order to keep the leg movement in harmony with the body movement, we make each leg move for 1/5 of a complete gait period T. Corresponding to Fig. 6, phase 2,3,5,6 is T/5, and phase 1,4 is T/10. From Eq.(4), we can see that the body has a forward component motion $d\frac{t}{T}$, which enables the body to move a distance d along the y direction in one period. So each leg also has to move a distance d in a period. Since each leg only takes $\frac{1}{5}T$ to move, the legs should move five times the speed of the body. The motion trajectory equation is as follows:

$$dx = 0$$
$$dy = 5d\frac{1}{T}(t - \frac{T}{40})$$
$$dz = h\sin\frac{5}{T}\pi(t - \frac{T}{40}) \tag{5}$$

where $t_{rh} \in (\frac{2T}{40}, \frac{10T}{40}), t_{rf} \in (\frac{10T}{40}, \frac{18T}{40}),$
$t_{lh} \in (\frac{22T}{40}, \frac{30T}{40}), t_{lf} \in (\frac{30T}{40}, \frac{38T}{40})$

The leg is not moving in the x direction. h is the sinusoidal amplitude, representing the maximum height of the lifting leg of the robot. t_{rh} represents the time for the right-hind leg to swing in one period. Same are with the other three legs. We have to pick the appropriate h to make sure that the leg is in the reachable space. The same goes for the other three legs.

III. Stability Margin Analysis

When the robot's dimensions and the step length are given, we can calculate the stability margin at any time for a given COG trajectory determined by the lemniscate of Gerono. In a gait cycle, the minimum stability margin is the main object we should pay attention to. In other words, the stability margin is greater than the minimum value throughout the walking cycle, and we will optimize this minimum value to optimize the stability margin of motion.

We can change the shape of the COG trajectory by changing the parameters a and b of the Gerono lemniscate in Eq. (3). And the stability margin is related to the shape of the COG curve. Also, the stability margin is related to the shape of the supporting triangle in the ground, i.e., it is also related to the step length of the robot.

A. The effects of parameters a and b

According to the Gerono lemniscate equation, parameter a determines the height of the curve, while parameter b determines the width of the curve, as shown in Fig. 9.

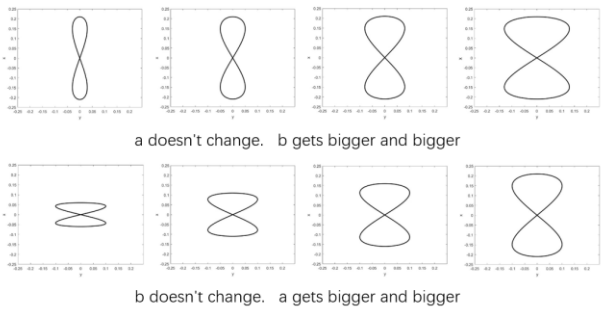


Fig.9. The influence of parameters a and b on the trajectory shape.

We discuss the influence of parameters a and b on the stability margin and their optimization process for Model 1. The four-leg stance phase is very stable and the its stability margin will not be discussed. When one leg swings and the other three are in contact with the ground, we assume that the positions of the three legs in contact with the ground are (x_1, y_1) (x_2, y_2) (x_3, y_3), and the projection of the COG on the ground is (x_0, y_0) .In this case, the formula of stability margin of the robot is as follows:

$$\begin{aligned} m_1 &= \frac{|(y_2-y_1)x_0+(x_1-x_2)y_0+(y_1-y_2)x_1+(x_2-x_1)y_1|}{\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}} \\ m_2 &= \frac{|(y_3-y_1)x_0+(x_1-x_3)y_0+(y_1-y_3)x_1+(x_3-x_1)y_1|}{\sqrt{(y_3-y_1)^2+(x_3-x_1)^2}} \\ m_3 &= \frac{|(y_3-y_2)x_0+(x_2-x_3)y_0+(y_2-y_3)x_2+(x_3-x_2)y_2|}{\sqrt{(y_3-y_2)^2+(x_3-x_2)^2}} \\ M &= \min\{m_1, m_2, m_3\} \end{aligned} \tag{6}$$

Model 1 is built based on actual Robot 1. Its body length is 0.6 m and its width is 0.4 m. If we assume that step length d is 0.05m, and firstly take an arbitrary pair of appropriate a and b to observe the change of stability margin in one period (a is 0.03, b is 0.05 in this case), we can calculate the stability margin during one gait cycle as shown in Fig.10.

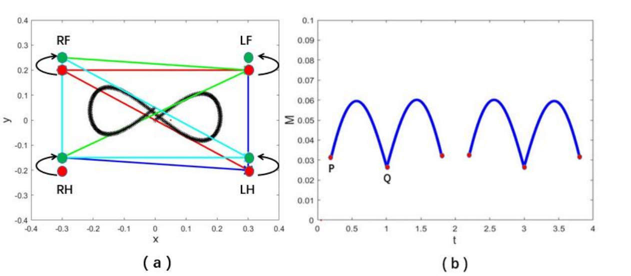


Fig.10. The actual trajectory of a complete period and the stability margin.

The trajectory of the COG and the supporting polygons at each stage are shown in Fig.10 (a). The red circle represents the initial position of the four feet. The green circle represents the position of the four feet after one period of movement. Since the order of the swinging leg is [RH, RF, LH, LF], the supporting triangles appear in order: [red, dark blue, green, and light blue] which is shown in Fig.10 (a) .By using Matlab, we can compute the value of the stability margin in one period, which is shown in Fig.10 (b). The local minimum points in the figure are P and Q. As we discussed above, now the problem of optimizing the stability margin of a period is transformed into optimizing P and Q. We need to select appropriate parameter a and b to maximize the minimum stability margin within a period. We take several pairs of a and b to analyze the stability margin curves, and the results are shown in Fig. 11

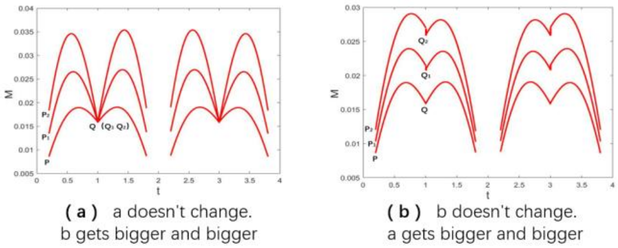


Fig.11. The influence of different a and b stability margin curve.

It can be seen from Fig. 11 that when a is fixed and b is gradually increased, point P moves upward significantly while point Q changes little or almost unchanged. However, when b remains unchanged and a gradually increases, it can be seen that point P moves upward without significant improvement, but at this point, point Q changes significantly. By setting $M = \min\{P, Q\}$ as the minimum stability margin over a period, we can get the relationship of M with respect to a and b, which is shown in Fig. 12. Take the appropriate a and b such that M has a maximum value, which is a convex problem according to the above analysis. So, we can find a pair of a and b to optimize M . it guarantees the minimum value of the stability margin to be maximized. We can get the current optimal in this case： $a = 0.211$, $b = 0.105$. The optimal stability margin is $M = 0.0883$；

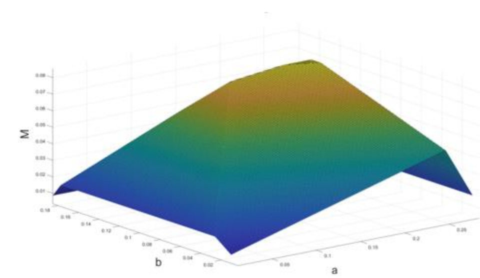


Fig.12. The minimum stability margin M with respect to a and b

B. *The influence of step length on stability margin*

The support polygon is composed of the support points between the foot and the ground, so different step sizes will not only affect the robot's speed but also change the shape of the support polygon, thus affecting the stability margin. Similarly, we expect the planner to be able to calculate the optimal step length in this gait according to the different configurations of the robot (simply represented as length and width), that is, moving at this step length can reach the local maximum of the minimum stability margin. In the working space of the robot's feet, the effect of step size on stability margin is a convex problem, too. Still using Model 1 for analysis. We have obtained the optimal a and b of the gait in this model. Apply this a and b and continue to explore the relationship between step length and stability margin. Through MATLAB, the relationship between the step length and the minimum stability margin of the robot's periodic motion is shown in Fig. 13.

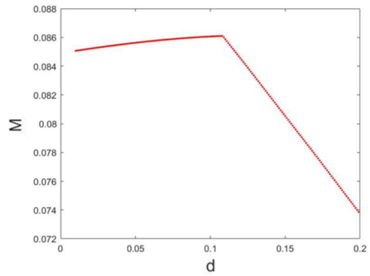


Fig.13. The relationship between the step length and the minimum stability margin.

The purpose of exploring the influence of step length on stability margin is not just to obtain the optimal step length. More importantly, this algorithm can balance the step length and stability margin according to demand. If the stability margin of our demand is already satisfied, then we can change the step size appropriately to control the speed of the robot.

C. Applications of algorithms on different robots

Another benefit of this gait algorithm is its generality, rather than just being designed for a particular robot. This also allows it to be applied to a variety of robots requiring high stability. When applied to different robots, the process of automatically calculating the optimal parameters a and b of the current robot is as follows:

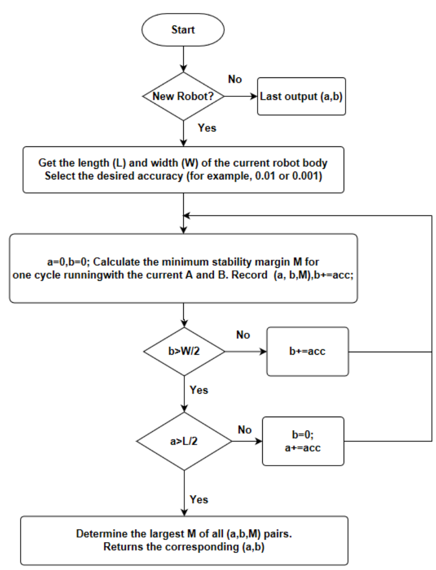


Fig. 14 Automatically obtain the optimal parameters.

If the algorithm is applied to the new quadruped robot, first of all, the body length and width of the current robot should be obtained, as well as the expected accuracy "acc". By setting the "acc", the error range of parameters a and b can be controlled. Besides, according to the geometry, we can find $a \in (0, \frac{L}{2})$ and $b \in (0, \frac{W}{2})$. By traversing the entire interval, we can get the maximum M, and then the optimal parameters a and b are obtained.

IV. Simulation Results

In order to verify the feasibility of the proposed method, we carried out simulation verification in the V-rep software with the Newton physics engine.

We will carry out simulations on the two simulation models respectively. Firstly, the correctness of the theory will be verified on Model 1, and the improvement of stability margin in robot motion will be visually demonstrated through a mass block experiment. Secondly, the gait algorithm will be applied to Model 2. Model 2 is a grasping robot that can perform a special task. Its body is equipped with a reconfigurable robotic arm and various sensors that create large deviations in the COG position with the geometric center. The application of the gait algorithm in Model 2 can verify the adaptability of the algorithm in different robots as well as the insensitivity of the algorithm to the position of COG.

A. Simulation results of the gait

The gait was verified in V-rep simulation software. The simulation result is shown in Fig.15. The quadruped robot walks to the right. The yellow curve is the trajectory of the COG. It can be seen that the trajectory of COG follows a periodic trajectory based on the Gerono lemniscate. Although the speed of the robot is not optimal, the stability of the robot is definitely better. Verification of stability is carried out later.

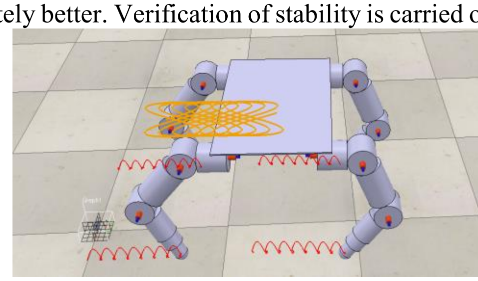
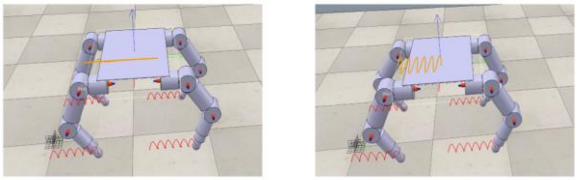


Fig. 15 Simulation results of the gait in Model 1

It should be noticed that in this gait, there are two special cases. For the two parameters a and b in the Gerono lemniscate, when one of the parameters is zero, they lead to two extreme cases. When parameter a is zero and parameter b is not zero, the COG trajectory degenerates into a straight line. The COG of the quadruped robot moves backwards and forwards in the straight line. The simulation result is shown in Fig.16(a). That's because the x-component of the robot's motion is zero. There's only an offset in the y-direction. Similarly, when a is not zero and b is zero, the COG trajectory degenerates into a standard sinusoidal curve as shown in Fig. 16(b). In both cases, the robot walks in a smooth and stable way.



(a) (b)

Fig. 16 Two extreme states of the gait algorithm

B. Stability optimization analysis

For the stability analysis, the previous section has made full theoretical analysis. This section is to observe whether the stability is optimized through the simulation software. The method is to add a mass block to the body of the quadruped robot. This test also takes into account a special task in real life, that is, for a quadruped robot transporting goods, which are small relative to the body, as the robot moves, the goods on the body may move relative to the body for some reason. Obviously, if the mass is distributed symmetrically along the geometric center of the robot body, no matter what the mass is, stability will not be affected. However, if the mass is in a corner of the body, then it has a relatively large effect on the stability of the robot.

Considering that this algorithm will also be implemented in subsequent practical tasks, the parameters of the simulation robot are similar to the real robot in all aspects. Each leg weighs 8 kg and the body 18 kg. The total weight of the quadruped robot is about 50 kg. The simulation parameters are the same as this.

The simulation results are shown on the left of Fig. 17. It can be seen that a cube with a side length of 0.2m was placed close to the left-front foot of the quadruped robot. When its mass is 20kg, it has almost no influence on gait stability.

This mass is 40% of the total mass of the robot, and the mass is located in the corner of the robot body. The excellent stability of this gait is well demonstrated.

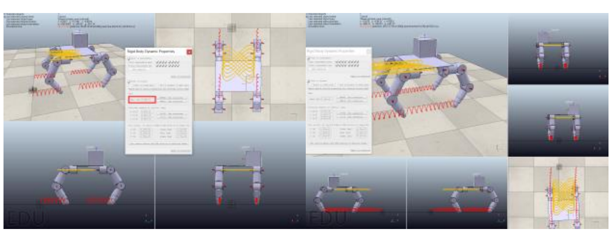


Fig. 17 Simulation results of model stability in V-rep.

We increase the weight of the block until the robot loses stability. It is found that 31 kg is the maximum acceptable weight of the block, which is 62% of the body mass of the robot. Beyond this point, the robot will tip over sideways. With a mass between 20-31kg, the robot will have poor stability in the process of movement, and its gait will be distorted but will not tip over. A long time of movement leads to a cumulative error that causes the robot to deflect in the direction of the mass, which is shown in the right of Fig. 17.

Finally, we extended the gait with omnidirectional motion. However, the design and tuning of parameters a and b under omnidirectional motion are different. The influence of step length on the stability margin should also be discussed further. We simply designed the parameters a and b, and specified the step length d_x and d_y. The simulation results in the simulation software are shown in Fig. 18. Our subsequent work will do more analysis on this omnidirectional gait.

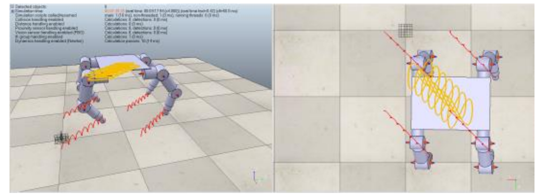


Fig. 18 The extension of the gait to omnidirectional walking.

C. The adaptability of this gait on different robots

All the parameters of simulation Model 2 are also established based on the real Robot 2. Due to the special task requirements of Robot 2, the manipulator and the sensors on the robot body makes the COG of the robot deviate from the geometric center to a large extent. The general gait algorithm is likely to cause problems in the stability of the robot during task execution. That's where our gait algorithm, which is insensitive to the center of gravity and significantly increases the stability margin, can be applied to.

For some major parameters, Model 2 has a body length of 0.5 m, a body width of 0.4 m, and a length of 0.3 m for both upper and lower legs. The algorithm can automatically calculate the parameters a and b that have the best stability margin under this gait for this robot, and they are: $a = 0.166$ and $b = 0.109$ respectively. For this robot, the minimum stability margin in a cycle is $M = 0.840$.

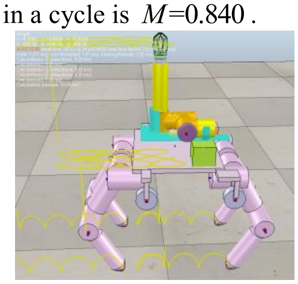


Fig. 19 Simulation results for Robot 2.

The simulation results in V-REP are shown in Fig. 19. It can be seen that the Model 2 robot can walk smoothly forward in this gait. In fact, robots can also cross obstacles or climb stairs in this gait. When crossing obstacles, due to the asymmetry of the load on the body, it requires very high stability for the robot. The robot could easily tip over in this situation. But our gait algorithm can successfully accomplish this task. Experiments are also taken on real Robot 2, which can walk very well (see the video attached). Therefore, the effectiveness of the proposed method is fully demonstrated.

V. Conclusions

In this paper, we propose a new method for the static gait planning of quadruped robots by using the lemniscate of Gerono to determine the COG motion. For different robot configurations, the two parameters of the Gerono lemniscate are the main tuning parameters. The algorithm can automatically solve the optimal parameters according to the sizes of the robots. On the other hand, the influence of the step length on stability margin is also discussed. According to the stability requirements of specific tasks, the planner may change the step length to obtain better stability. This gait method has been proved to improve the stability of static gait significantly.

In the V-REP simulation environment, the effectiveness of the proposed gait algorithm was verified. The mass block simulation on Robot 1 shows that the proposed algorithm can improve the stability of the robot. Meanwhile, in another simulation, the algorithm is applied to the robot Model 2. This shows the adaptability of the algorithm to different robots. It is also proved that the gait algorithm is insensitive to the COG position of the robot. In the future, it is expected that this gait can be applied to more complicated environments to improve the adaptability of the robot. At the same time, the motion in any direction in the horizontal plane is simulated simply, which shows the robot can achieve omnidirectional walking in this gait. More analysis on the omnidirectional walking will be taken in the future.

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